Dielectric Sleeve Resonator Techniques for Variable-Temperature Microwave Characterization of Ferroelectric Materials

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Abstract– Low-loss sleeve resonators can be used for accurate microwave dielectric characterization of rod-shaped test specimens. The test specimen is inserted into the dielectric sleeve resonator and placed centrally in a metal cavity. With the use of additional sleeve resonators having differing external diameters or permittivities, a single specimen can be characterized at multiple frequencies. Sleeve resonators can also be employed for accurate dielectric characterization of high- permittivity specimens having dielectric loss factors greater than 0.001. Closed-form solutions for TE_{0np} resonant mode structure are given. Uncertainty relations for permittivity and dielectric loss are also shown, which demonstrate that when sample electric energy filling factors are greater than 0.4, relative uncertainties in measured permittivity and dielectric loss tangent are less than 1% and 4%, even for relative permittivities greater than 600. Example measurements are given that illustrate how this dielectric resonator system can be employed for dielectric characterization of ferroelectric materials at temperatures both near or far from their Curie temperatures.

I. INTRODUCTION

Use of a specimen as a dielectric resonator is a commonly used approach for accurate complex permittivity evaluation of low-loss materials [1-5]. Dielectric resonator techniques, when applicable, generally provide higher accuracies than other resonator methods because specimen partial electric energy filling factors for appropriately chosen mode structure and practically sized samples are greater.

Low-loss, temperature-stable dielectric ring resonators can be used advantageously for accurate dielectric measurements of cylindrical specimens at microwave frequencies. First, the ring resonator controls the nominal resonant frequency at which dielectric characterization of an inserted rod specimen is performed. Thus, with the use of several low-loss ring resonators, the dielectric properties of a *single* specimen may be evaluated at several discrete frequencies. Second, if the specimen's loss characteristics are not known and are relatively high, this approach often permits an accurate loss determination since the unloaded Q-factor of the composite system may be measurable when that of the specimen as a single dielectric resonator is not. Third, smaller test specimens are required at low frequencies for characterization. The placement of the loaded sleeve resonator system in a cylindrical cavity permits ready variable-temperature dielectric characterization of the rod specimen provided the dielectric characteristics of the sleeve resonator and conductive losses of the enclosing cavity have been accurately determined.

II. THEORY FOR DIELECTRIC SLEEVE RESONATOR IN CAVITY

The dielectric sleeve resonator system is illustrated in Fig. 1. The sample has a complex relative permittivity $\epsilon^*_{r,s} = \epsilon'_{r,s} - j\epsilon''_{r,s} = \epsilon'_{r,s}(1 - j\tan\delta_{e,s})$, where $\tan\delta_{e,s}$ is the dielectric loss tangent of the specimen. The sleeve resonator is characterized by $\epsilon^*_{r,slv} = \epsilon'_{r,slv} - j\epsilon''_{r,slv} = \epsilon'_{r,slv}(1 - j\tan\delta_{e,slv})$. In general, the electromagnetic wave equation to be solved in cylindrical coordinates, subject to boundary conditions, is given by

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial F}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 F}{\partial \phi^2} + \frac{\partial^2 F}{\partial z^2} + k^2 F = 0, \quad (1)$$

where $k = \omega\sqrt{\mu_0\epsilon_0\epsilon'_r}$ is the wavenumber in the medium, $k_r = \sqrt{k^2 - \beta^2}$ or $k_r = \sqrt{\beta^2 - k^2}$ is the radial wavenumber, β is the z-directed propagation constant, $\omega = 2\pi f$ is the radian frequency, ϵ_0 and μ_0 are the free space permittivity and permeability (8.854×10^{-12} F/m and $4\pi\times10^{-7}$ H/m), and F represents either the axial electric field, E_z (for TM- or E-modes), or the axial magnetic field, H_z for (TE-or H-modes). Once E_z and H_z are obtained as solutions of the wave equation, all other components can be ob-

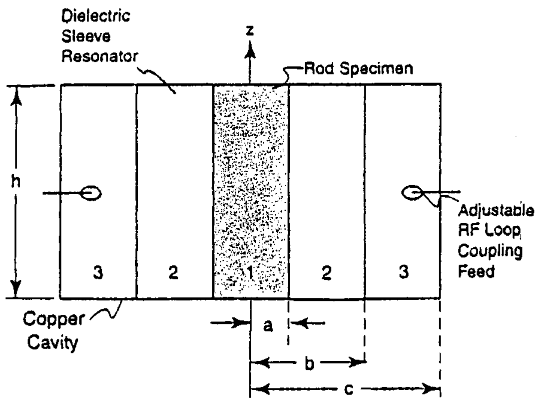

Figure 1: Dielectric sleeve resonator system for testing rod specimen with relative complex permittivity $\epsilon^*_{r,s} = \epsilon'_{r,s}(1 - j\tan\delta_s)$. Sleeve resonator has inner radius a, outer radius b, and complex permittivity $\epsilon^*_{r,slv} = \epsilon'_{r,slv}(1 - j\tan\delta_{e,slv})$.

tained from Maxwell's equations,

$$\begin{aligned} E_r &= \frac{1}{k_r^2}\left(\frac{\partial^2 E_z}{\partial r\partial z} - \frac{j\omega\mu_0}{r}\frac{\partial H_z}{\partial\phi}\right), \\ E_\phi &= \frac{1}{k_r^2}\left(\frac{1}{r}\frac{\partial^2 E_z}{\partial\phi\partial z} + j\omega\mu_0\frac{\partial H_z}{\partial r}\right), \\ H_r &= \frac{1}{k_r^2}\left(\frac{\partial^2 H_z}{\partial r\partial z} + \frac{j\omega\epsilon_0\epsilon_r^*}{r}\frac{\partial E_z}{\partial\phi}\right), \\ H_\phi &= \frac{1}{k_r^2}\left(\frac{1}{r}\frac{\partial^2 H_z}{\partial\phi\partial z} - j\omega\epsilon_0\epsilon_r^*\frac{\partial E_z}{\partial r}\right). \end{aligned} \quad (2)$$

Typically, the TE modes are most useful in dielectric measurements because there is no capacitive coupling between either the sleeve resonator or dielectric specimen and end plates of the cavity. The enclosing cylindrical cavity also allows minimal dimensional changes in the positions of the upper and lower cavity end plates in variable temperature measurements that could potentially affect TE resonant mode structure in the composite dielectric resonator system. the following, both the dielectric sleeve resonator and under test are isotropic.

A. Permittivity Evaluation

For axi-symmetric TE_{0np} resonant modes, the only non-zero components of the electromagnetic field in each region are H_z, E_ϕ, and H_r. Equations (1) and (2) can be solved for the field components in each region. Continuity of the tangential electric and magnetic fields at $r = a$ and $r = b$, as well as vanishing of E_ϕ at $z = 0$, $z = h$, and $r = c$ leads to the following characteristic equation for TE_{0np} modes,

$$\frac{k_{r1}J_0(k_{r1}a)J_1(k_{r2}a) - k_{r2}J_1(k_{r1}a)J_0(k_{r2}a)}{k_{r2}J_1(k_{r1}a)Y_0(k_{r2}a) - k_{r1}J_0(k_{r1}a)Y_1(k_{r2}a)} + \frac{k_{r2}J_0(k_{r2}b) - k_{r3}A(b)J_1(k_{r2}b)}{k_{r2}Y_0(k_{r2}b) - k_{r3}A(b)Y_1(k_{r2}b)} = 0, \quad (3)$$

where

$$A(b) = -\frac{K_0(k_{r3}b) + CI_0(k_{r3}b)}{K_1(k_{r3}b) - CI_1(k_{r3}b)}, \quad (4)$$

$$C = \frac{K_1(k_{r3}c)}{I_1(k_{r3}c)}, \quad (5)$$

$J_0, J_1$and Y_0, Y_1 are Bessel functions of the first and second kinds, K_0, I_0 and K_1, I_1 are modified Bessel functions, and $k_{r1}^2 = k_0^2\epsilon'_s - \beta^2$, $k_{r2}^2 = k_0^2\epsilon'_{slv} - \beta^2$, $k_{r3}^2 = \beta^2 - k_0^2$ and $\beta = p\pi/h, p = 1, 2, \cdots$. The first zero of eq(3) for $p = 1$ corresponds to TE_{011} mode resonance for a given aspect ratio $2a/h$ of the sample, inner and outer diameters $2a$ and $2b$ of the sleeve resonator of height h, enclosing cavity diameter $2c$, and a specimen real relative permittivity $\epsilon'_{r,s}$. This equation may be used for predicting TE_{0mp} resonant frequencies for a loaded sleeve dielectric resonator or for evaluating permittivity of inserted rod specimens when identified TE_{0mp} frequencies, sleeve permittivity, and sleeve and rod specimen geometries are known.

B. Dielectric Loss Tangent

The dielectric loss tangent of the specimen under test is evaluated from the realtion,

$$\frac{1}{Q_0} = \frac{1}{Q_s} + \frac{1}{Q_p}, \quad (6)$$

where Q_0 is the unloaded Q-factor, $Q_s = 1/(p_{e,s}\tan\delta_{e,s})$ is the sample quality factor, and parasitic losses Q_p^{-1} are given by

$$\frac{1}{Q_p} = p_{e,slv}\tan\delta_{e,slv} + \frac{1}{Q_c}, \quad (7)$$

with $p_{e,slv}$ being the electric energy filling factor of the sleeve resonator and the subscript e denoting electric filling factor and loss. Exact expressions are derivable for the field components requisite to evaluating specimen loss tangent and the partial electric field energy filling factors for specimen and sleeve, $p_{e,s}$ and $p_{e,slv}$. The conductor losses are given in terms of the cavity endplate and wall losses and is given by

$$\begin{aligned} Q_c^{-1} &= \frac{P_{endplates} + P_{wall}}{\omega_0 W} \\ &= \frac{R_s}{G}, \end{aligned} \quad (8)$$

where $P_{endplates}$ and P_{wall} denote power losses in the cavity endplates and sidewall, W is the total electric

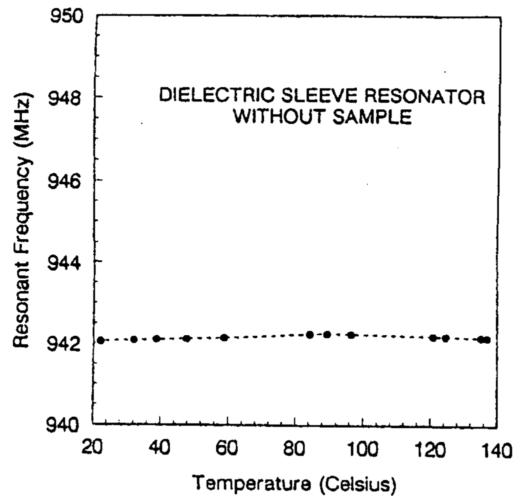

Figure 2: Resonant frequency of dielectric sleeve resonator as function of temperature without sample insertion.

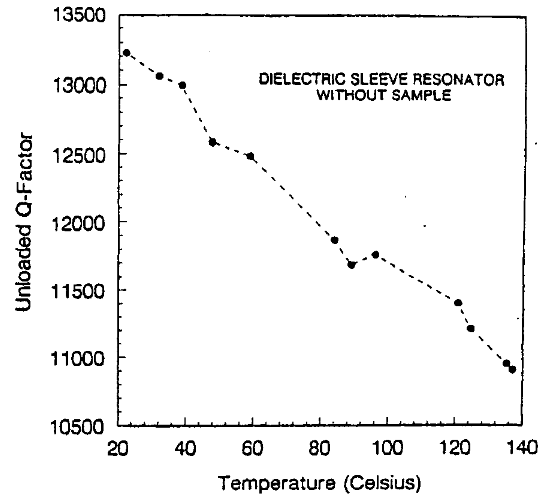

Figure 3: Unloaded Q-factor of dielectric sleeve resonator system as function of temperature without sample insertion.

field energy in resonant system at radian resonant frequency ω_0, R_s is the measured metal surface resistance, and G is the geometric factor factor of the resonant system.

III. MEASUREMENTS

A low-loss, commercially available dielectric sleeve resonator was fabricated and placed inside a metal cavity for variable- temperature measurements. The TE_{011}-mode resonant frequency of the system without specimen insertion is determined by the permittivity and aspect ratio of the dielectric ring resonator. The permittivity and dielectric loss tangent of a rod specimen is then evaluated by measuring the differences in resonant frequency and unloaded Q-factors for the TE_{011} mode with and without specimen insertion. The entire resonant system is then placed into an environmental chamber. The average time needed to attain ± 1 °C temperature stability is 2 hours for each measurement temperature.

Variable-temperature measurements were first made at UHF frequencies on the dielectric sleeve resonator without sample insertion. These measurements, taken between 20 °C and 140 °C are shown in Figs. 2 and 3. The sleeve resonator has a real relative permittivity and dielectric loss tangent of 37.1±0.2 and $3.24\times10^{-5}\pm2\times10^{-5}$, respectively, at 20°C. The internal and external diameters of the sleeve are 10 mm and 60 mm and the height of the sleeve was 45 mm. The diameter of the cavity was 100 mm.

The sleeve resonator exhibited a temperature coefficient with respect to resonant frequency, τ_f, that was close to 0 ppm/°C. The measured unloaded Q-factor of the empty dielectric sleeve resonator decreases as temperature increases and must be taken into account in measuring dielectric losses of ferroelectric specimens. As temperature increases for the ferroelectric specimens tested, the resonant frequency shift decreases and unloaded Q-factor increases. This occurs because dielectric measurements are being taken further from the Curie temperature that defines the ferroelectric/paraelectric state of the tested specimen. Evaluated real relative permittivities and dielectric loss tangents for these example materials are shown in Figs. 4 and 5. In Fig. 5 the variable-temperature and variable-frequency conductor losses were taken into account, as was the variable-temperature characteristics of the sleeve resonator. The rms relative uncertainty in evaluation of the specimen's real permittivity, assuming negligible gaps between sample and sleeve and negligible uncertainties in the height of specimen and sleeve, is given by

$$\frac{\Delta\epsilon'_{r,s}}{\epsilon'_{r,s}} = [(S_b^{\epsilon'_{r,s}}\frac{\Delta b}{b})^2 + (S_c^{\epsilon'_{r,s}}\frac{\Delta c}{c})^2 + (S_{\epsilon'_{r,slv}}^{\epsilon'_{r,s}}\frac{\Delta\epsilon'_{r,slv}}{\epsilon'_{r,slv}})^2 + (S_f^{\epsilon'_{r,s}}\frac{\Delta f}{f})^2]^{1/2}, \quad (9)$$

where the weight sensitivity functions of the respective relative uncertainties in b, c, $\epsilon'_{r,slv}$, and f are given by

$$S_x^{\epsilon'_{r,s}} = \frac{x}{\epsilon'_{r,s}}\frac{\partial\epsilon'_{r,s}}{\partial x}. \quad (10)$$

The rms relative uncertainty in the specimen's dielectric loss tangent may similarly be given in terms of the relative uncertainties of the sample and sleeve partial electric filling factors, the surface resistance of the enclosing cavity, the geometric factor, the loss tangent

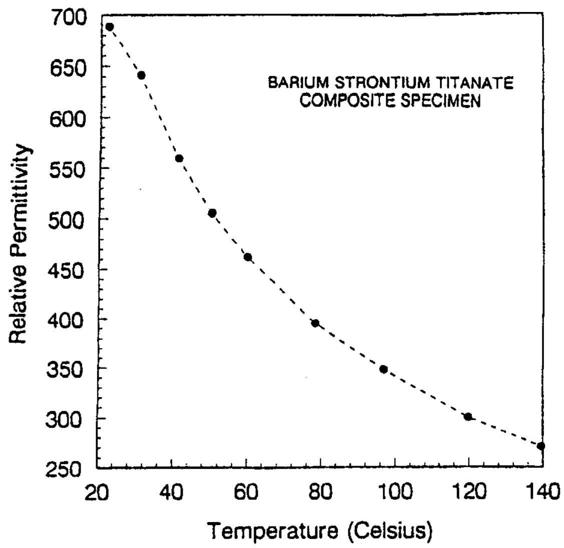

Figure 4: Computed real permittivity of rod specimen as a function of temperature.

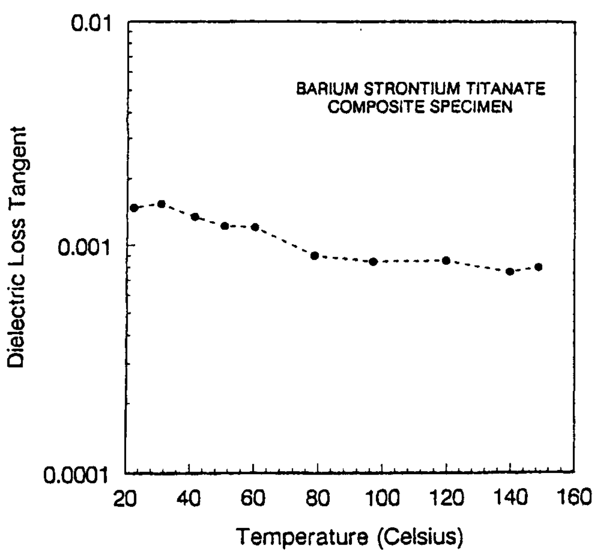

Figure 5: Computed dielectric loss tangent of rod specimen as a function of temperature.

of the sleeve resonator, and the system unloaded Q-factor. The uncertainty in the diameter of the sleeve resonator and real permittivity of the sleeve resonator dominate the uncertainty in the measured permittivity of the specimen. Total uncertainties in dielectric loss tangent are dominated by uncertainties in measured unloaded Q-factor, sample partial electric energy filling factor, and to a lesser extent, the uncertainties in sleeve resonator loss tangent and cavity conductor losses for $\epsilon'_{r,s}$ greater than 300.

IV. SUMMARY

Low-loss dielectric sleeve resonators can be used for accurate TE_{011} dielectric property measurements of high-permittivity materials. Use of multiple sleeve resonators or higher-order resonant modes permit wider bandwidth characterization of the same specimen. The nominal measurement frequency is controlled by the sleeve dimensions and permittivity. For specimen fractional partial electric energy filling factors greater than 0.5, relative uncertainties in permittivity are less than 1%, even for relative permittivities as high as 1000, with typical dimensional uncertainties as high as 0.01 mm and with an uncertainty in the sleeve permittivity of 0.5%. Uncertainties in dielectric loss tangent critically depend on the uncertainties with which the variable-temperature and variable-frequency sleeve loss tangent and surface resistance of the enclosing cavity are determined, as well as the uncertainty in the unloaded Q-factor.

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